

The black disk and the dip in the differential elastic cross section at asymptotic energy

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Abstract

We test the validity of the black disk limit in elastic scattering by studying the evolution of the dip in the scaling variable $\tau = -t_D \sigma^{tot}$, where t_D is the transverse momentum squared at the dip and σ_{tot} the total cross section. As $s \rightarrow \infty$ and $-t_D \rightarrow 0$, τ may consistently be approaching the black disc value, $\tau \xrightarrow{\sqrt{s} \rightarrow \infty} \tau_{BD} = 35.92 \text{ GeV}^2 \text{ mb}$.

Recent results from LHC (pp scattering at 7 TeV) and from Auger Observatory (pAir at 57 TeV) on total and elastic cross sections, [1-3], may be quite relevant to improve our understanding of the asymptotic behavior, $\sqrt{s} \rightarrow \infty$, of cross-sections.

There are two important theorems obtained by making use of fundamental concepts as analyticity, crossing symmetry and unitarity:

- 1) Froissart bound [4],

$$\sigma(s)^{tot} \sim 2\pi R^2(s) \sim \log^2(s/s_0). \quad (1)$$

The proof of the theorem requires the existence of a maximum angular momentum $L(s)$, proportional to some radius $R(s)$, above which the contributions to the partial wave sum are negligible.

- 2) Geometric Scaling GS [5,6,7] In the limit of Froissart behavior, (1), it

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follows that

$$ImF(s, t) = ImF(s, 0)\varphi(\tau), \quad (2)$$

where $ImF(s, t)$ is the imaginary part of the amplitude and φ an entire function of the scaling variable τ ,

$$\tau \equiv -t\sigma^{tot}. \quad (3)$$

One should notice that t and the impact parameter b are conjugate variables with the result that β ,

$$\beta^2 = b^2/\sigma^{tot}, \quad (4)$$

is also a scaling variable. GS ideas and phenomenology were developed in [6] and [7].

One should also notice that the original GS does not agree with data. The ratio σ^{el}/σ^{tot} is predicted to be constant while a clear growth with energy is seen in data [8].

In order to see why is it so, let us write (see, for instance, [9]):

$$\sigma^{tot}(s) = 2\pi \int db^2 ImG(s, b) \xrightarrow{GS} 2\pi R^2(s) \int_0^1 d\beta^2 ImG(\beta) \quad (5)$$

and

$$\sigma^{el}(s) = \pi \int db^2 [ImG(s, b)]^2 \xrightarrow{GS} \pi R^2(s) \int_0^1 d\beta^2 [ImG(\beta)]^2, \quad (6)$$

where $G(s, b)$ is the elastic amplitude and in (6) the real part was neglected. From (5) and (6) one immediately sees that $\sigma^{el}/\sigma^{tot} = const \leq 1/2$.

The relevant cross- sections, (5) and (6), contain explicit dependence on energy via $R^2(s)$, the quantity controlling the size and range of the interactions. But energy should also affect the quark- gluon matter density, showing evolution towards saturation. We introduce a second function depending on energy, $f(s)$, to describe evolution of matter density.

We shall next make a grey disk approximation, and identify the averaged in β of $ImG(\beta)$ with $f(s)$;

$$\langle ImG(\beta) \rangle \simeq f(s), \quad (7)$$

with

$$\frac{df}{ds} \geq 0, \quad (8)$$

and

$$f(s) \xrightarrow{s \rightarrow \infty} 1. \quad (9)$$

Equation (9) is a consequence of unitarity saturation in the black disk limit. Note that (8) says that blackness increases with energy.

Making use of (5), (6) and (7) we obtain:

$$\sigma^{el}(s)/\sigma^{tot}(s) = \frac{1}{2}f(s), \quad (10)$$

in violation of GS, and

$$\sigma^{tot}(s) = 2\pi R^2(s)f(s). \quad (11)$$

Asymptotically, (10) and (11) satisfy GS. Note that the function $f(s)$, describing unitarity saturation, was introduced in [11]:

$$f(s) = 1 - e^{-\bar{\Omega}(s)}, \quad (12)$$

the opacity $\bar{\Omega}(s) = 2(\gamma_1 + \gamma_2 \ln(s) + \gamma_3 \ln^2(s))$, and $\gamma_1 = 0.29$, $\gamma_2 = 0.0191$, $\gamma_3 = 0.0013352$ to keep common notations with [10]. In both cases, (10) and (11), the asymptotic behavior, as energy increases, is reached from below (see [12]).

The physics of (10) and (11), in the $f(s) \rightarrow 1$ limit, is black disk physics. In (10) we obtain

$$\sigma^{el}(s)/\sigma^{tot}(s) \rightarrow \frac{1}{2}. \quad (13)$$

In (11), having in mind that for the black disk $B(s, t = 0) \rightarrow R^2/4$, where B is the slope parameter we arrive at

$$\sigma^{tot}/B(s, 0) \rightarrow 8\pi \quad (14)$$

Relations (13) and (14), see [13], are well known black disk relations.

Making use of the analytical properties of amplitudes and cross- sections it was possible to estimate the ratio $\sigma^{inel}(s)/\sigma^{tot}(s)$ at asymptotic energies to obtain a value (0.509 ± 0.011) [14], consistent with the naive expectation for a black disk (see [15] for general discussion). Our neglect of $ReG(\beta)$, in particular in the forward peak, is a way of having, asymptotically, the black disk.

We turn next to GS and write, see (2) and (3)

$$\frac{d\sigma}{dt}(t)/\frac{d\sigma}{dt}(0) \xrightarrow{\sqrt{s} \rightarrow \infty} \varphi^2(\tau), \quad (15)$$

where τ is the scaling variable. If GS was exact (13) would be exact. If it is just true asymptotically we have to concentrate in the limit $\sqrt{s} \rightarrow \infty$.

In order to test GS let us consider the evolution of the position of the minimum $\tau_D = -t_D \sigma^{tot}$, seen in the range $\sim (20 \text{ GeV} \leq \sqrt{s} \leq 7 \text{ TeV})$. One observes that σ^{tot} increases with energy (see (10), $\sigma^{tot} \sim R^2(s)f(s)$) so one needs $-t_D$ to decrease with energy. As we do not have a strict prediction for the evolution of $-t_D$ we write, for instance, $-t_D \sim \frac{1}{\sigma^{tot}}$ and we have GS for any value of \sqrt{s} .

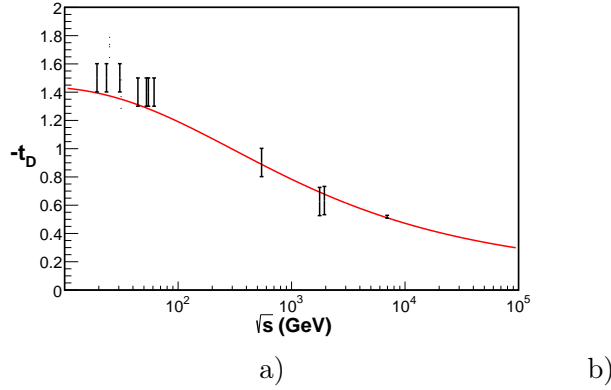


Figure 1: $-t_D$ as a function of energy \sqrt{s} . Here $R(s)$ is given by the parametrization $R(s) = R_0 \ln(s/s_0)$, with $R_0 = .0936 \text{ mb}^{1/2}$, and $\sqrt{s_0} = 2.216 * 10^{-9} \text{ GeV}$, (16).

As GS can only be correct asymptotically we write

$$-t_D = \frac{1}{2\pi R^2(s)} \frac{1}{f(s)^\alpha} \tau_{BD} \quad (16)$$

with $\sigma^{tot}(s)$, given by (11), and $\tau_{BD} = 35.92 \text{ GeV}^2 \text{ mb}$ [16] being the black disk τ and α is a parameter. If $\alpha = 1$ GS works at all energies. Experimentally we obtained $\alpha = 1.47$, and GS is asymptotic.

In Fig.(1) we present the obtained energy dependence of $-t_D$, (16). In Fig. (2) we show $\tau = -t_D \sigma^{tot}$ as a function of \sqrt{s} . τ seems to approach $\tau_{BD} = 35.92 \text{ GeV}^2 \text{ mb}$, in a slow process. The star (*) in Fig. (2) corresponds to our expectation for $\sqrt{s} = 14 \text{ TeV}$, using information from Fig. (1) At the star $\tau_* = 44.9$ ($\sqrt{s} = 14 \text{ TeV}$). In conclusion, we find at present LHC energies indications that we are approaching black disk behavior ($\sigma^{el}/\sigma^{tot} \rightarrow 1/2$, $\sigma^{tot}/B(s) \rightarrow 8\pi$, and $\tau_{DIP} \rightarrow \tau_{BD} = 35.92 \text{ GeV}^2 \text{ mb}$. However we are still far from asymptopia.

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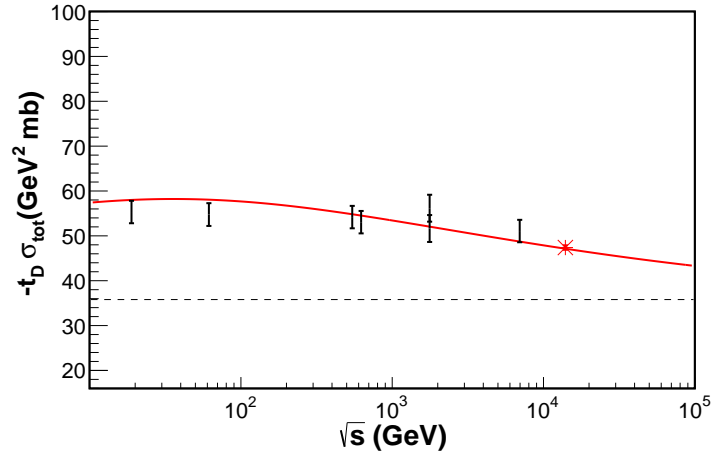


Figure 2: Solid line shows $-t_D \sigma_{tot}$ as function of energy from (11) and t_D as in previous figure. The dashed line shows the black disk limit. The star corresponds to expectation for $\sqrt{s} = 14$ TeV.

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